ON THE CAPACITY OF ALOHA PACKET RADIO NETWORKS FOR LOCAL TRAFFIC MATRICES *

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Abstract

If we have a large population of users sharing a common broadcast channel using the Slotted Aloha access scheme, we know that the maximum achievable throughput is only 36% $(1/e)$ of the channel capacity, independent of the exact number of users of the channel. If we allow the packet radio units to restrict their range so that they transmit at a power which just allows their communication partner to receive the message, we find that we can achieve a much higher throughput (depending on the traffic matrix) by *spatial reuse* of the channel. In this paper we consider the extreme case of packet radios wishing only to communicate with their 'nearest neighbor' and find both analytically and by simulation that we can achieve a throughput which is a *linear* function of the number of users in the system. This behavior is attributable to a reduction in interference due to spatial separation and suggests that, by forcing the packet radio units to use reduced power, we may be able to achieve similar throughputs for less restricted traffic matrices.

1. INTRODUCTION

One of the major problems in effective utilization of computer resources is the distribution of those resources to the user. This problem has been greatly alleviated by the advent of communication networks but local distribution still remains a problem. The concept of broadcast packet radio for local access was first utilized in the ALOHA system [ABRA 70] and more recently, the Advanced Research Projects Agency of the Department of Defense has undertaken a project to investigate the use of more general broadcast packet radio systems [KAHN 77]. A packet radio network consists of many packet radio units (PRUs)** sharing a common radio channel such that when one unit transmits, many other units will hear the packet, even though it is addressed to only one of them. This feature, inherent in broadcast systems, in conjunction with the fact that we have no control over access to the channel, results in destructive interference when several packets are received simultaneously.

Many studies have been made on the capacity (maximum achievable throughput) of centralized single hop communication networks using broadcast radio as the communication medium. In [LAM 74] we find an extensive analysis for the fully connected one hop slotted ALOHA access scheme and in [TOBA 74, KLEI 75b] we find similar results for Carrier Sense Multiple Access (CSMA).

A class of problems, which has received little attention in the literature, is that of point to point (i.e., non-centralized) networks. In this paper we will be concerned with point to point communication. The simplest approach that allows realization of an arbitrary traffic matrix is to let all nodes be within range of each other. The capacity under these circumstances will be the same as for the centralized network, $1/e$. If, however, we restrict the range of the packet radio units so that they are just able to reach their destination (i.e., a one-hop transmission), we find a reduction in the amount of interference generated and are, thus, able to achieve higher throughputs.

The interference pattern and hence the network performance will therefore depend on the traffic requirements. If the traffic requirement is for nodes at opposite extremes of the network to communicate, we will have the fully connected network that has a capacity of $1/e$. If all of the traffic is local, however, we are able to perform at much higher throughput levels by spatial reuse of the channel. In this paper we are concerned with determining the capacity for this local traffic situation and attempt to find the 'best' traffic matrix (i.e., that which will allow the maximum capacity). The problem of finding the 'best' matrix is hard, so instead we find some simple upper and lower bounds on the capacity and then look at the performance for some specific local matrices which have excellent performance. These traffic matrices allow us to achieve a throughput which is a *linear function of the number* of nodes in the network (a significant improvement over the fixed $1/e$ of the fully connected network !!). The enormous capacity improvement that this allows, suggests that restriction of transmission range may be beneficial in true (connected) networks which are able to support an *arbitrary* traffic requirement.

Elsewhere, we have studied two different system designs for realization of an arbitrary traffic matrix where this range restriction is indeed beneficial. In [KLEI 78] we investigated multi-hop communication and were able to construct networks having a capacity that is proportional to the square root of the number of nodes in the network. We also found that an optimum transmission radius exists which maximizes the system capacity.

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** All components of the packet radio network (terminals, computers or repeaters) use a common device for channel access. This device is called the packet radio unit.

In [SILV 79] we restrict our attention to single-hop communication and are able to achieve a capacity which is proportional to the logarithm on the number of nodes in the network.

The networks that we study in this paper are random in nature, that is to say, the nodes are uniformly distributed throughout the area under consideration (which may be a circular disc for two dimensional networks or a line segment in one dimension). Having generated the network in this fashion (which can be thought of as either representing an arbitrary network or a snapshot of a mobile one), we proceed to impose a matrix of traffic requirements, and then determine what transmission radii are necessary to support this communication. Knowing the transmission radii we can determine the interference pattern and compute the throughput for heavy traffic. This heavy traffic throughput is considered to be the capacity of the network.

In the following section we give an example of how this capacity is computed for a simple example.

2. COMPUTATION OF CAPACITY

Figure 1 shows a random network of four nodes, the circles drawn around each node representing the area covered by a transmission of that node.

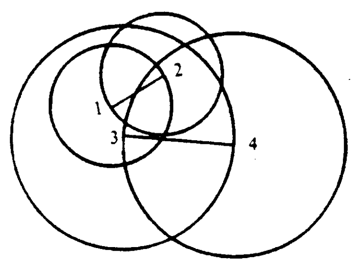

Figure 1 A Simple Four Node Network

The transmission radii shown are determined by the traffic matrix, which in this example requires that nodes 1 and 2 are a communicating pair and that 3 and 4 are the other pair.

In order to find the capacity we consider the heavy traffic situation, that is, when all nodes are always busy and have something to transmit. The probability of a successful reception, s_i, at node i can then be computed.

$$s_i = \Pr\{\text{partner of } i \text{ transmits}\} \Pr\{i \text{ does not transmit}\} \Pr\{\text{no one else whom } i \text{ hears, transmits}\} \tag{1}$$

If p_i denotes the probability that node i transmits in any slot, then the probability of a successful reception at node 1 is:

$$s_1 = p_2(1-p_1)(1-p_3) \tag{2}$$

For heavy traffic in the Slotted ALOHA mode, the capacity of node i, γ_i, is equal to the probability of a successful reception, s_i.

If we assign a transmission probability of ½ to each node the throughputs are:

$$\gamma = \begin{pmatrix} (1/2)^3 \\ (1/2)^2 \\ (1/2)^4 \\ (1/2)^3 \end{pmatrix} = \begin{pmatrix} \frac{1}{8} \\ \frac{1}{4} \\ \frac{1}{16} \\ \frac{1}{8} \end{pmatrix} \tag{3}$$

This gives a total throughput, γ $(=\sum \gamma_i)$, of:

$$\gamma = \frac{9}{16} \tag{4}$$

3. DEFINITIONS

In the following discussion we refer to several variables defined as follows:

The Density λ represents the average number of points (nodes) per unit area.

The Number of Nodes in the network will be n.

The Traffic Matrix τ defines the amount of flow between nodes in the network. In this paper it will be used to identify the traffic pattern and is assumed to be normalized in some fashion. As each node will only be communicating with one other node in the system, each row (and column) of this matrix will have exactly one non-zero element.

$$t_{ij} = \begin{cases} 1 & \text{if } i \text{ talks } j \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

For the particular arrangement shown in the example above (Fig 1) the, traffic matrix is:

$$\tau = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{6}$$

The Adjacency Matrix **A** defines the hearing graph, i.e., which nodes can hear which others. This matrix is not necessarily symmetric as the underlying graph is directed since different nodes may be using different transmission radii.

$$a_{ij} = \begin{cases} 1 & \text{if } j \text{ hears } i \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

For the earlier example (Fig. 1), the adjacency matrix is:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \tag{8}$$

The Hearing Distribution H_i defines the probability that a node is in range of i other nodes. We call this the excess number of nodes heard, as we count neither the node itself nor its partner. H_i is the probability that **A** has $i+2$ ones in any column.

$$H_i = \Pr\{\text{a node hears } i \text{ excess nodes}\} \tag{9}$$

The Hitting Distribution h_i represents the probability that i nodes hear a given node's transmission. It corresponds to the probability of **A** having $i+2$ ones in any row.

$$h_i = \Pr\{\text{an excess of } i \text{ nodes hear when a node transmits}\} \tag{10}$$

The Transmission Probability vector **P** is an extremely important system parameter, which defines the probability with which a node will transmit in any slot.

$$p_i = \Pr\{\text{node } i \text{ transmits in any slot}\} \tag{11}$$

In this paper we will use two different policies for the retransmission probability. The first corresponds to the optimum value for a pair of non-interfering nodes and uses a probability of ½ for all nodes. The other policy is to attempt to set the local traffic load to unity and thus reduce overloading in the local channel. In [LAM 74, ABRA 70, YEMI

?8] we find that if a node hits an excess of k nodes (as in the definition of the hitting and hearing distributions), then the optimum transmission probability is $\frac{1}{k+2}$

The Success Probability vector s represents the probability of any node receiving a packet successfully in any slot.

$$s_i = \Pr\{\text{node } i \text{ successfully receives a packet}\} \tag{12}$$

The Throughput γ. For heavy traffic the throughput is identical to the success probability ($\gamma_i = s_i$). The system capacity, γ, is the sum of the nodal throughputs.

$$\gamma = \sum \gamma_i \tag{13}$$

We use γ^j to represent the throughput for a j-dimensional model.

4. GENERAL MODEL

With the above definitions we can write the success probability in terms of the other variables. Suppose that i and j are a pair of communicating nodes, i.e., $t_{ij} = 1$.

$$s_i = \Pr\{j \text{ transmits}\} \Pr\{i \text{ does not transmit}\} \Pr\{\text{none of } i\text{'s neighbors transmits}\}$$

$$= p_j (1-p_i) I \tag{14}$$

where

$$I = \Pr\{\text{no interference}\} \tag{15}$$

Since the networks that we consider are homogeneous, the throughput for all nodes is identically distributed; we can therefore drop the subscripts corresponding to the particular node under investigation. As noted before, the retransmission probability is only dependent on the number of nodes hit by a transmission. If we make the further assumption that both nodes of a partnership hit the same number of nodes* we obtain the following expression for the throughput:

$$\gamma = I \sum_{k=0}^{n-2} h_k \frac{1}{k+2} \left[1 - \frac{1}{k+2}\right] \tag{16}$$

Assumption: We assume that the interference heard by any node is independent of the number of nodes that he hits. With this assumption we can proceed with the computation of I.

$$I = \sum_{k=0}^{n-2} H_k (1-q)^k \tag{17}$$

where q is the expected transmission probability of a node that you hear. Defining the z-transform of the hearing distribution:

$$H(z) = \sum_{k=0}^{n-2} H_k z^k \tag{18}$$

we can rewrite the expression for I in terms of this transform $H(z)$:

$$I = H(1-q) \tag{19}$$

The expected transmission probability is given by:

$$q = \sum_{k=0}^{n-2} \theta_k \frac{1}{k+2} \tag{20}$$

where θ_k are the 'adjusted hit probabilities', i.e., the probability that a node you hear hits k excess nodes when he transmits. We cannot use the hit probabilities as defined above, since a node is much more likely to hear a node that hits many other nodes than one that hits only a few.

* As both nodes are transmitting at the same range, the *expected* number hit by a transmission will be the same.

$$\theta_k = \begin{cases} 0 & k = 0 \\ ckh_k & k \geq 1 \end{cases} \tag{21}$$

where c is a normalization constant such that $\sum \theta = 1$.

We have thus reduced the problem of finding the throughput to that of determining the sets of probabilities h_i and H_i.

Below we try to find the best traffic matrix; first, however, we look at some bounds.

5. SIMPLE BOUNDS ON PERFORMANCE

In this section we give simple upper and lower bounds on the performance for the best possible traffic matrix (BTM).

5.1 Upper Bound

If there were no interference between pairs of nodes, we would be able to achieve a performance equal to that obtainable by $n/2$ independent pairs. One independent pair is able to support a throughput of ½ (which is achieved for a transmission probability of ½ [ABRA 70]. Thus,

$$\gamma_{BTM} \leq \frac{n}{4} \tag{22}$$

5.2 Lower Bound

As a lower bound we consider how many pairs (clean pairs) we can support without any of them causing interference to any other pairs. Consider a pair of nodes in the network, P and Q. If these are to communicate without causing any interference, Q must be P's nearest neighbor and P must also be Q's nearest neighbor.

5.2.1 One Dimension

In Figure 2 we show two points in a one-dimensional random network. There are n nodes randomly located in the unit line. For simplicity we approximate this by a Poisson process of density λ $(=1/n)$.

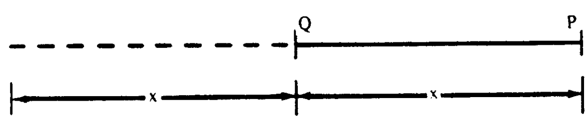

Figure 2 No Interference in One Dimension

Suppose Q is P's nearest neighbor. The distribution of x ($\overline{PQ}$) is given by [KEND 63]:

$$f(x)dx = 2\lambda e^{-2\lambda x} dx \tag{23}$$

For no interference we require that there be no point closer to Q than P. That is, there is to be no point on the dashed line. The length of this line is clearly equal to x, and the probability of finding no point there, r, is:

$$r = e^{-\lambda x} \tag{24}$$

So the probability that a point is a member of a clean pair, g, is:

$$g = \int_0^\infty 2\lambda e^{-2\lambda x} e^{-\lambda x} dx$$

$$= \int_0^\infty 2\lambda e^{-3\lambda x} dx = \frac{2}{3} \tag{25}$$

We see, then, that we can find a traffic matrix which can support $.67n/2$ clean pairs, which will allow a throughput of $.67n/4$. Thus,

$$\frac{.57n}{4} \leqslant \gamma^{1}_{BTM} \leqslant \frac{n}{4} \qquad (26)$$

5.2.2 Two Dimensions

We now consider the two dimensional analog.

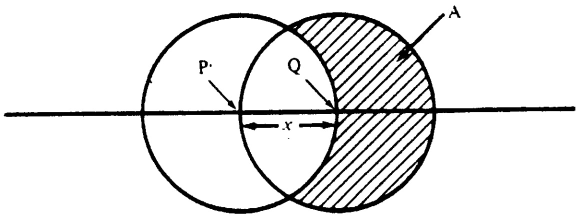

Figure 3 No Interference in Two Dimensions

Let Q be P's nearest neighbor, we assume that P and Q are randomly located in the unit circle by a Poisson process of parameter λ. (This model is not exact, as in fact we place precisely n points in a unit circle but for large n it is a good approximation.) The distribution, $f(x)$, of $\overline{PQ}$ is given by [KEND 63, ROAC 68]:

$$f(x)dx = 2\pi\lambda x e^{-\lambda\pi x^2} dx \qquad (27)$$

For no interference we require that there be no point closer to Q than P. That is, there is to be no point in the shaded area, A, encircling Q (there is no point in the circle around P since Q is the nearest neighbor). This area can be found to be:

$$A = x^2 \left[\frac{\pi}{3} + \frac{\sqrt{3}}{2} \right]$$

$$= 1.913x^2 \qquad (28)$$

The probability of finding no one in this area is $e^{-\lambda A}$. So the probability that a point is a member of a clean pair, g, is:

$$g = \int 2\lambda\pi x e^{-\lambda A} e^{\lambda\pi x^2} dx$$

$$= \frac{\pi}{\pi + 1.913}$$

$$= 0.622 \qquad (29)$$

This result can also be found in [DEWI 77].

Thus we can find a traffic matrix allowing a throughput of $.62n/4$ and this gives us the following bounds:

$$\frac{.62n}{4} \leqslant \gamma^{2}_{BTM} \leqslant \frac{n}{4} \qquad (30)$$

In figure 4 we show simulation results for the number of clean pairs in a random network (for one- and two-dimensions) and also plot the values predicted by the model. As expected, the agreement between the two is excellent.

6. CASE STUDIES

We have found bounds for the performance of the 'best traffic matrix', but determination of the optimal is a difficult (NP-complete) problem. We choose, therefore, to look at some specific connection strategies allowing us to achieve high throughputs. For some of these cases we can proceed with the analysis outlined earlier, but in all cases we give

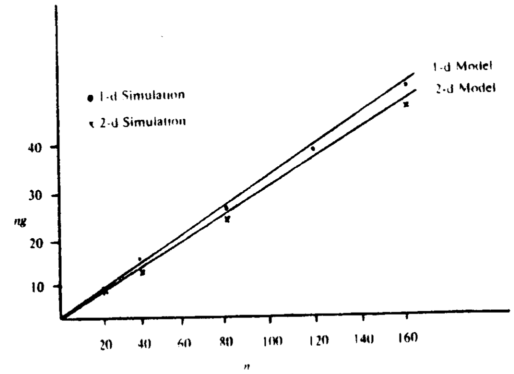

Figure 4 Clean Pairs - 1-d and 2-d

simulation results.

6.1 Nearest Unpaired Neighbor (NUN) in Two Dimensions

In this section we consider random two-dimensional networks and give a low interference connection strategy, the behavior of which falls within the bounds given in section 5.

The connection policy for the nearest unpaired neighbor scheme is as follows:

1) Generate the random network (even number of nodes); mark all nodes as unpaired.
2) Find the two closest unpaired nodes and connect them; mark them as paired.
3) If all nodes are paired, we have finished; otherwise, return to step 2.

The traffic pattern generated by this algorithm is satisfied by giving each node sufficient power to exactly reach his destination. We show a sample network generated in this fashion in figure 5 (both dashed and solid lines represent pairing).

The next step is to assign transmission policies as outlined in section 3. The weighted scheme assigns each node a transmission probability of $\frac{1}{k+2}$, where k is the excess number of nodes hit by a transmission. We show simulation results for this scheme in figure 6, which also shows the bounds given by equation (30). We have also included the curves for the fully connected ALOHA system [ABRA 70] and for a random traffic matrix satisfied by exact transmission radii [SILV 79]. We see that the performance of the weighted system appears to be linear and exceeds the lower bound. Although we do not analyze this scheme here, its performance is almost identical to that of one-dimensional adjoining, which is analysed in section 6.3.

The second approach that we use in trying to find high throughput is *expurgation*. Each node is assigned a transmission probability of ½ and then those pairs causing excessive interference (starting with the longest link in the network, as this is the one most probably causing the greatest inteference) are not allowed to transmit (expurgated). If this process is continued until no further improvement is found, we have the optimally expurgated scheme, the performance of which is also shown in figure 6. We notice that the behavior is very similar to the weighted transmission scheme. In figure 5 the dashed lines indicate those pairs which would be expurgated.

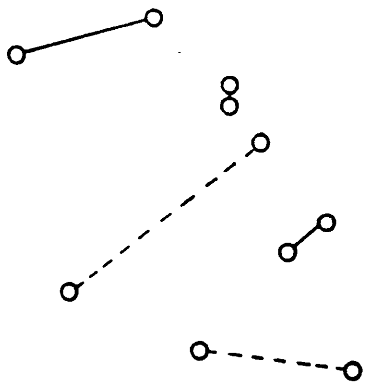

Figure 5 A 10 Node Random Network

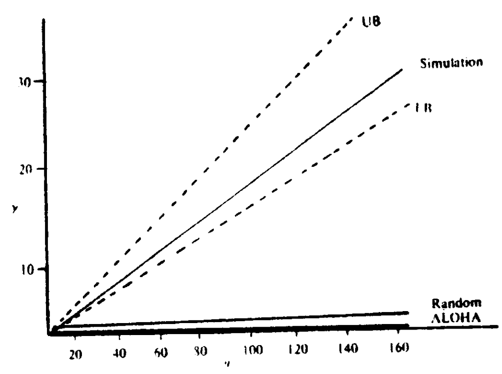

Figure 6 NUN in Two Dimensions

In figure 7 we show the effect of expurgation on the throughput for various network sizes. In all cases the throughput increases until about 16% of the nodes are no longer communicating. As we delete additional pairs of nodes, the throughput decreases linearly to zero since the pairs being expurgated are, in fact, not causing interference.

6.2 Nearest Unpaired Neighbor (NUN) in One Dimension

This is the one-dimensional equivalent of the two-dimensional scheme outlined above. In the following section we consider a simpler version of this, in which every node is connected (adjoined) to his left (or right) neighbor. We find that the performance of NUN and the adjoining scheme are almost identical and therefore do not show the results explicitly for this scheme (NUN), but rather investigate the simpler (Adjoining) scheme in greater detail.

6.3 Adjoining in One Dimension (ADJ)

For this scheme we randomly locate n points on the unit line and then connect adjacent pairs starting from one end.

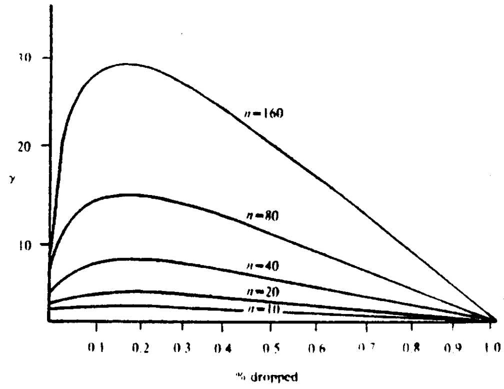

Figure 7 Expurgation for 2-D NUN

6.3.1 Determination of the Hitting Distribution h

We know the distribution of the distance to the neighbor on your left (or right) and we must determine how many points are expected to fall in this distance on the other side of the connection, this being the number of points that will hear you. The distribution of the neighbor distance, x, is:

$$f(x)dx = \lambda e^{-\lambda x}dx \tag{31}$$

The points that you hit are precisely those that fall in a distance x on your right (left). (Notice that this is not the same as the distribution for your *nearest* neighbor as given in equation (27).) The number of points falling in this distance on the other side is Poisson distributed, thus:

$$\Pr\{i \text{ in a distance } x\} = e^{-\lambda x}\frac{(\lambda x)^i}{i!} \tag{32}$$

So,

$$h_i = \int_0^\infty \frac{(\lambda x)^i}{i!} e^{-\lambda x}\lambda e^{-\lambda x}dx \quad = \int_0^\infty \lambda^{i+1} x^i e^{-2\lambda x}dx$$

$$= (1/2)^{i+1} \tag{33}$$

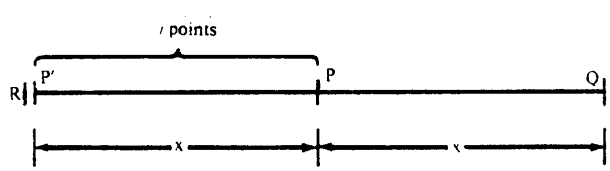

Figure 8 Hitting Distribution for 1-d Adjoining

We can derive this distribution in an alternate manner without having to rely on the exponential or Poisson distributions as follows. Suppose that node P (whose partner is Q) hits i excess nodes. Let R be the first point on the left that cannot hear P, for this to happen, P must be to the right of the midpoint of $\overline{QR}$, the probability of this event is 1/2. If x is the distance from P to Q then consider a point P′ at a distance x to the left of P. Now all the i excess points must fall to the left of the midpoint of $\overline{QP'}$ (i.e. to the left of P). The probability of this event is $(1/2)^i$. Thus the probability that P hits i points is:

$$[illegible] = [illegible] \qquad (34)$$

From this we can determine [illegible]

$$[illegible] = \frac{[illegible]}{2^{[illegible]}} \qquad (35)$$

Summing these to obtain c we get:

$$\sum [illegible] = 1 - \frac{n-2}{2^{n-1}} \qquad (36)$$

$$\Rightarrow \quad c = \frac{2^{n-1}}{2^{n-1}-n+2} \qquad (37)$$

$$\approx 1 \qquad \text{(for large } n\text{)} \qquad (38)$$

From this we may determine the expected transmission probability of interfering nodes, q:

$$q = c \sum_{i=0}^{n-2} \frac{i}{2^{i+1}} \frac{1}{i+2}$$

$$= c \left[\sum_{i=0}^{n-2} (½)^{i+1} - 4 \sum_{i=2}^{n} \frac{1}{i} (½)^{i} \right]$$

$$= c \left[1 - (½)^{n-1} + 4 \left(\log(½) + ½ + \sum_{i=n+1}^{\infty} \frac{1}{i} (½)^{i} \right) \right]$$

$$\approx 3 - 4\log(2) \qquad \text{(for large } n\text{)} \qquad (39)$$

In order to proceed we must find the hearing distribution.

6.3.2 Determination of the Hearing Distribution H

Figure 9 Hearing Distribution

Suppose that between P and Q' there are $k-1$ points. In order for P to hear Q' (the partner of Q) all k points (the $k-1$ intervening points and Q' itself) must fall to the left of midpoint of $\overline{PQ}$. The probability of this is easily determined to be:

$$\Pr\{\text{P hears Q}\} = (½)^k \qquad (40)$$

We will say that Q' is at distance k from P if there are $k-1$ intervening points. We find that the points who interfere with P from the right are at distances 1,3,5,7, etc.

We may use an identical argument for points on the left of P and we find that these points are at distances 2,4,6 etc. Let us therefore call the event of being hit by a person of distance k away E_k. Then:

$$\Pr\{E_k\} = (½)^k \qquad (41)$$

If P hears j people this means that exactly j of the set of events $\{E_k\}$ have occurred. We can therefore write the expressions for H_j. Let us first look at the probability that P does not hear any interference (this is H_0).

$$H_0 = \Pr\{\text{P hears only his partner}\}$$

$$= \Pr\{\text{none of } E_i \text{ occur}\}$$

$$= \prod_{i=1}^{\infty} \left[1 - (½)^i \right] \qquad (42)$$

Unfortunately it appears that this product does not have a closed form. It is in fact related to the inverse of the partition function. We use the following identity of Euler to evaluate this expression, which converges extremely rapidly and also gives us a bound on the error (as it is an alternating monotonically decreasing series).

$$\prod_{k=1}^{\infty} (1 - x^k) = \sum_{j=-\infty}^{\infty} (-1)^j x^{\frac{j(3j+1)}{2}} \qquad (43)$$

We find:

$$H_0 \approx 0.289 \qquad \text{(for large networks)} \qquad (44)$$

To find the probability that P hears one additional point we must find the probability that exactly one of the E_k occurs, and generalizing, if P hears j additional points then exactly j of the set $\{E_k\}$ must occur.

$$H_1 = \sum_{k=1}^{n-1} \frac{H_0}{1-(½)^k} \qquad (45)$$

So, in general:

$$H_j = \sum_{k_1=1}^{n-1} \sum_{k_2=k_1+1}^{n-1} \cdots \sum_{k_j=k_{j-1}+1}^{n-1} \frac{H_0}{[1-(½)^{k_1}][1-(½)^{k_2}] \cdots [1-(½)^{k_j}]} \qquad (46)$$

These have been evaluated by computer and the values are shown in Table 1 (for a 100 node network).

number j	H_j analytical	H_j simulation	h_j analytical	h_j simulation
0	0.289	0.302	0.500	0.504
1	0.464	0.446	0.250	0.250
2	0.209	0.212	0.125	0.117
3	0.036	0.037	0.063	0.068
4	0.003	0.002	0.031	0.030
5	0	0	0.016	0.014
6	0	0	0.008	0.009
7	0	0	0.004	0.004

Table 1 Hearing and Hitting Distributions for 1-d ADJ

From these we can evaluate I, the expected interference.

$$I = \sum_{j=0}^{n-2} H_j (1-q)^j$$

$$\approx 0.78924 \qquad (47)$$

This gives the throughput for node j, γ_j:

$$\gamma_j = I \sum_{i=0}^{n-2} h_i \frac{1}{i+2} \left[1 - \frac{1}{i+2} \right]$$

$$= 2I \left[\sum_{i=2}^{n} \frac{1}{i} (½)^i - \sum_{i=2}^{n} \frac{1}{i^2} (½)^i \right]$$

$$\approx 2I\left[\log(2) + \frac{[\log(2)]^2}{2} - \frac{\pi^2}{12}\right] \qquad \text{(for large } n\text{)}$$

$$\approx .1756 \tag{48}$$

Thus the total network throughput, γ, is given by:

$$\gamma \approx .176n \qquad \text{(for large } n\text{)} \tag{49}$$

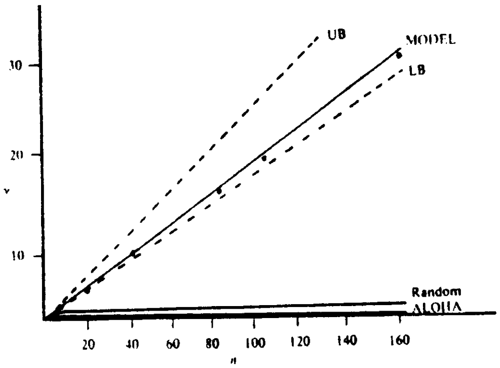

Figure 10 Throughput for ADJ in One Dimension

Figure 10 shows the throughput predicted by this model and also the bounds found in equation (26). We also show simulation results for both the weighted and expurgation schemes. We see very good agreement between analytical and simulation results. For reference purposes we plot the fully connected ALOHA capacity [ABRA 70] and the capacity for a random traffic matrix satisfied by exact transmission radii, a model for which can be found in [SILV 79].

6.4 An Overview

All of the schemes of the previous sections have very similar performance; we were able to obtain an analytic expression for the one-dimensional case. All of them exhibit linear behavior in excess of the lower bound with slopes of approximately $1/2e$. We feel that these schemes are in fact very close to the best possible traffic matrix, although we do not have any concrete justification for this statement at this time.

7. CONCLUSIONS

In this paper we have looked at the capacity of Packet Radio Networks for local traffic. In particular we were trying to determine what traffic pattern would allow us to achieve the highest throughput. We were to able to find some simple bounds on the performance of the the 'best' traffic matrix. We found that the total throughput γ for the optimal traffic matrix is bounded in one dimension by $.67n/4 \leqslant \gamma \leqslant n/4$ and in two-dimensions by $.62n/4 \leqslant \gamma \leqslant n/4$. We found that local traffic seemed to give low interference and so studied some specific configurations in more detail. For these local configurations we were able to achieve a capacity which is a linear function of the number of nodes in the network, exceeding the lower bound. We analyzed the one-dimensional case, showing $\gamma \approx .7n/4$. In all cases our simulation results supported our analytic findings.

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